1. a) $Y(s) = G_p(s)\, G_c(s)\, \cancel{R(s)} = \frac{G_c(s)}{s+1} \frac{10}{s}$

need pole at -2 no have $\tau = 0.5$

let $G_c(s) = \frac{(s+1)K}{s+2}$

$$Y(s) = \frac{1}{s+2} \frac{K10}{s}$$

$$y_{ss} = \lim_{s \to 0} sY(s) = \frac{K10}{2}, \quad \text{so} \quad K = 2$$

$$G_c(s) = \frac{(s+1)2}{s+2}$$

b) $\frac{Y(s)}{R(s)} = \frac{1}{s+.5} \frac{(s+1)}{s+2} 2$

τ is governed by the slowest pole
so $\tau = 1/.5 = 2$ sec

$$Y(s) = \frac{(s+1)2}{(s+.5)(s+2)} \frac{10}{s} \qquad \text{so} \quad y_{ss} = 20$$

$$\Rightarrow e_{ss} = r_{ss} - y_{ss} = -10$$

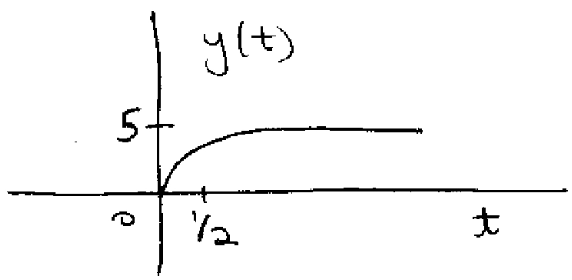
2. a) $\dfrac{Y(s)}{R(s)} = \dfrac{G_c G_p}{1 + G_c G_p}$

$$= \frac{K_p/s+1}{1 + K_p/s+1} = \frac{K_p}{s+1+K_p}$$

pick $K_p = 1$

b). $y_{ss} = \lim_{s \to 0} sY(s) = \dfrac{10}{2}$ so $e_{ss} = r_{ss} - y_{ss} = 5$

c)

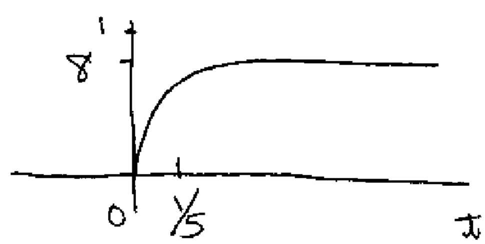


d) $K_p = 4 \implies \dfrac{Y(s)}{R(s)} = \dfrac{4}{s+5}$

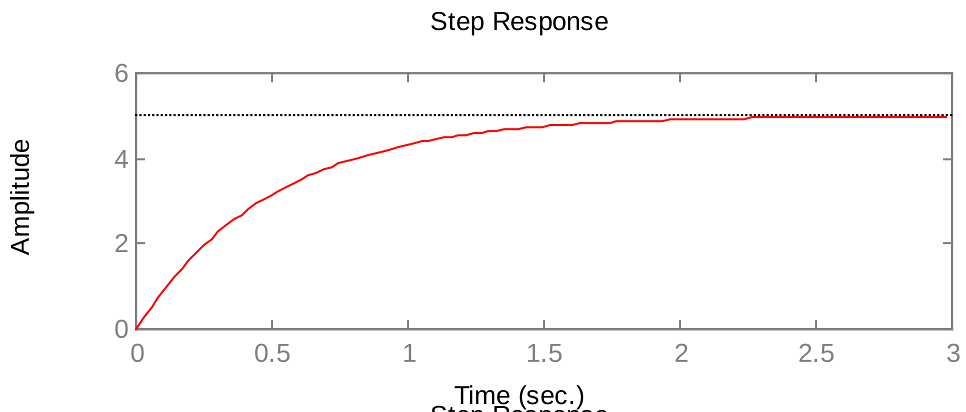
$$\tau = 1/5 \text{ sec}$$

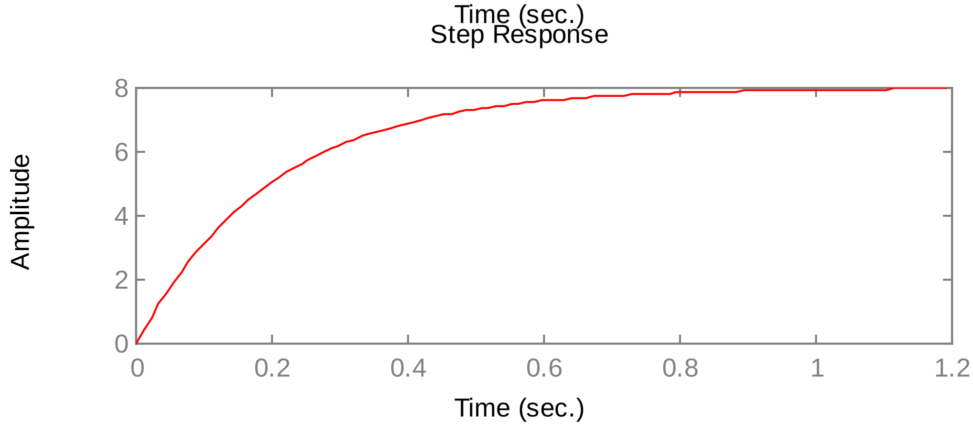
$$y_{ss} = \lim_{s \to 0} sY(s) = \frac{4}{s+5}(10)\Big|_{s=0} = 8$$

$e_{ss} = 2$



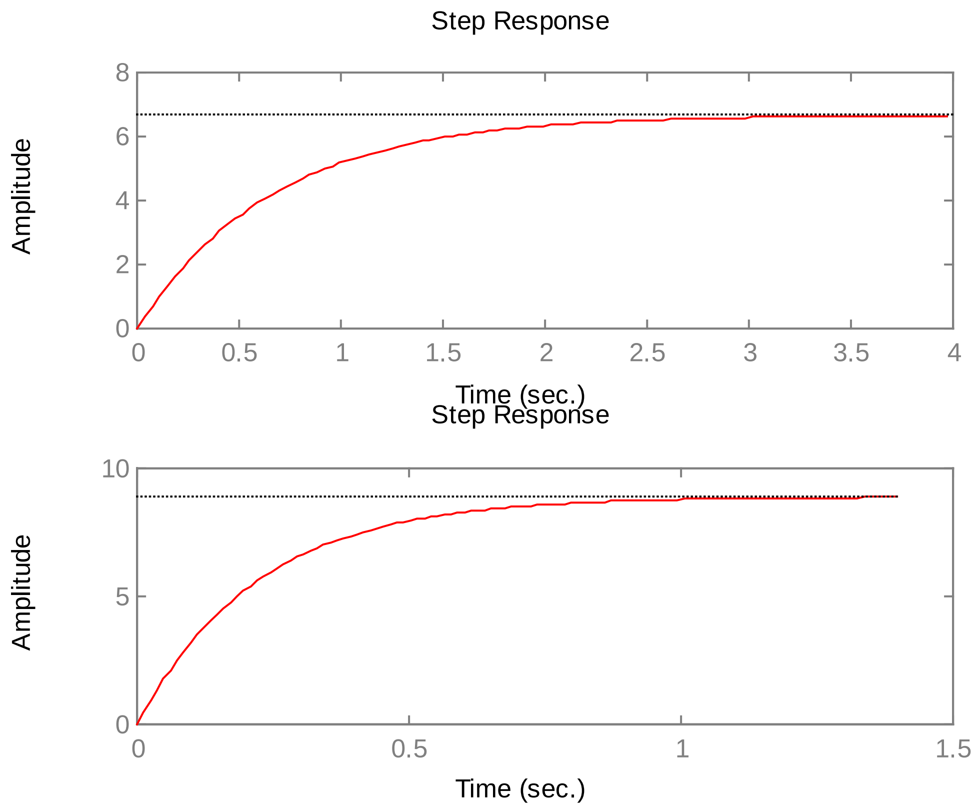
e) The plots below are the closed loop step responses for the controllers defined in a) and d) (with the nominal plant). Top plot is for $K_P = 1$ and the bottom plot is for $K_P = 4$.





f) The actual closed loop system is represented by $G_{cl}'(s) = \dfrac{G_c G_p'}{1 + G_c G_p'} = \dfrac{K_P}{1 + K_P + 0.5}$

where G_P' represents the actual plot. The closed loop step responses are shown for K_P=1 (top plot) and K_P=4 (bottom plot).



3. $G_p = \frac{1}{s+0.5}$, let $R(s) = \frac{R_0}{s}$ $\quad (r(t) = R_0 u(t))$

$$Y(s) = \frac{4}{s+5} \frac{R_0}{s}$$

$$y_{ss} = \frac{4R_0}{5} = 10 \Rightarrow R_0 = \frac{50}{4} = \frac{25}{2}$$

Apply $r(t) = \frac{25}{2} u(t)$ to actual plant

$$Y(s) = \frac{4}{s+4.5} \frac{25}{2s}$$

so $y_{ss} = \frac{50}{4.5} = 11.11$

$$e_{ss} = r_{ss} - y_{ss} = -1.11$$

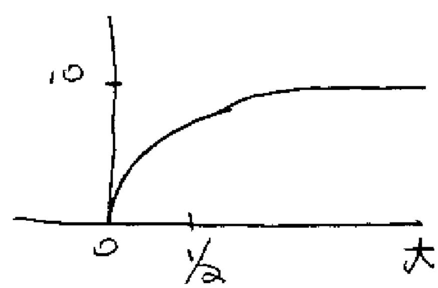
4. $G_c = \frac{K_p(s+1)}{s}$

$$G_{cl} = \frac{G_c G_p}{1+G_c G_p} = \frac{K_p/s}{1+K_p/s} = \frac{K_p}{s+K_p}$$

a). $K_p = 2$

b). $Y(s) = \frac{K_p}{s+K_p}\frac{10}{s}$, if $K_p=2$ $\quad y_{ss} = 10$

so $e_{ss} = r_{ss} - y_{ss} = 0$

c). 

d)

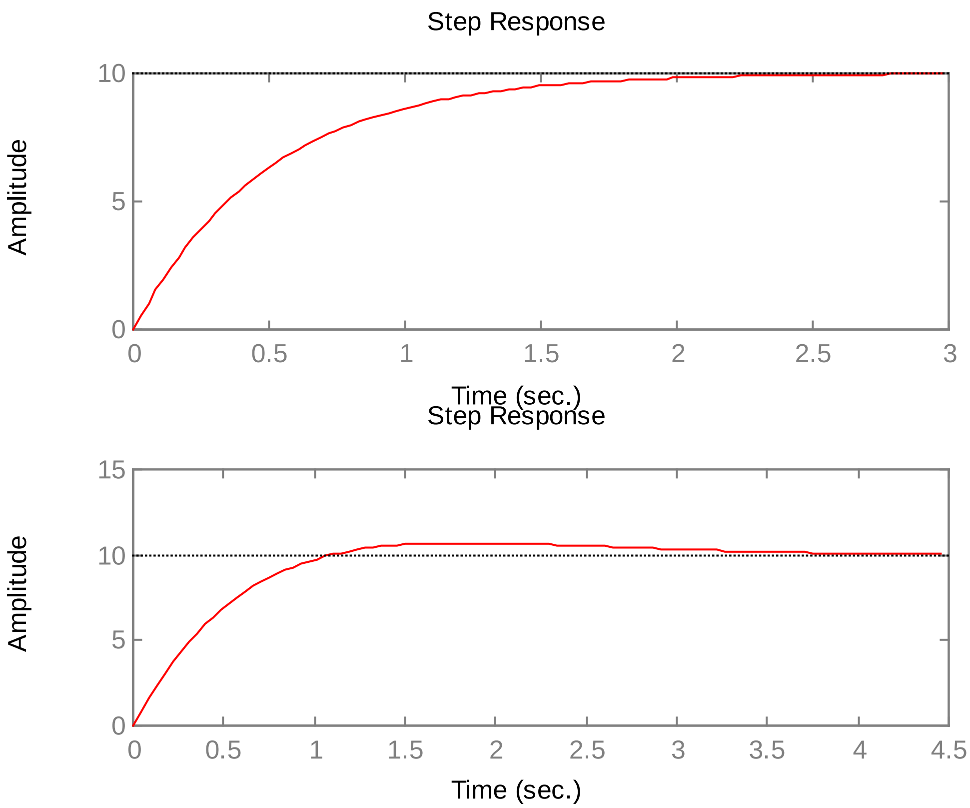
```
num= 2;
den = [1 2];
step (num * 10, den)
title ('PI for nominal plant')
```

(see plot next page)

e) $$G_{cl}' = \frac{G_c G_p'}{1+G_c G_p'} = \frac{\frac{2(s+1)}{s}\frac{1}{s+.5}}{1+\frac{2(s+1)}{s(s+.5)}} = \frac{2(s+1)}{s^2+2.5s+2}$$

```
num = [2 2]; den = [1 2.5 2];
step (10* num, den)
```

f) The following plots show the closed loop step response for the nominal plant with the PI compensator (top plot) and the actual plant with the PI compensator (bottom plot).



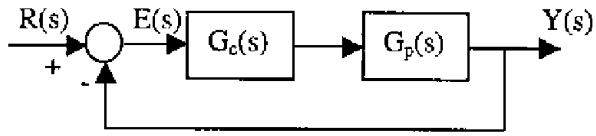
5. Comparison of closed loop results:

	e_{ss} (nominal plant)	τ (nominal plant)	e_{ss} for actual plant	τ (nominal plant)
Open Loop	0	0.5	-10	2
P with K_P = 1	5	0.5	3.33	0.667
P with K_P = 4	2	0.2	1.11	0.22
P with open loop adj.	0	0.2	-1.11	0.22
PI	0	0.5	0	0.8

For nominal behavior, the open loop, the P with larger value of K_P (especially with the open loop adjustment in r(t)) and the PI all work well. All can be made faster by proper choice of the controller parameters.

You can get a measure of robustness by noting how sensitive the controller is to the change in the plant. For example, how close is the behavior of the closed loop system with the actual plant to that of the closed loop system with the nominal plant. The open loop controller is very sensitive with very large changes in τ and e_{ss} . The P controller with larger gain is less sensitive than the P with smaller gain (the percent change in the τ and e_{ss} are lower). The PI controller is not sensitive at all with respect to steady-state error, but is somewhat sensitive to the time constant.

6



$$G_p(s) = \frac{0.1}{s+0.1}$$

$$G_{cl} = \frac{G_c G_p}{1 + G_c G_p} = \frac{\frac{.1 K_p}{s+.1}}{1 + \frac{.1 K_p}{s+.1}} = \frac{.1 K_p}{s + .1 + .1 K_p}$$

i) $\tau = \frac{1}{.1 + .1 K_p} \le 5 \Rightarrow K_p \ge 1$

ii) $Y(s) = G_{cl} R(s) = \frac{.1 K_p}{s + .1 + .1 K_p} \frac{1}{s}$

$e_{ss} = 1 - y_{ss} \le .05$, so $y_{ss} \ge 0.95$

$$y_{ss} = \lim_{s \to 0} s\,Y(s) = \frac{.1}{.1 + .1 K_p} = \frac{1}{1 + K_p} \ge 0.95$$

$\Rightarrow K_p \ge 19$

this satisfies both criteria

7

i) $G_c G_p = \frac{4}{s}$, $G_{cl} = \frac{G_c G_p}{1 + G_c G_p} = \frac{4}{s+4}$

$\tau = 1/4$, $e_{ss} = r_{ss} - y_{ss} = 1 - 1 = 0$ (PI control on a step input)

pole at -4 is real & has good relative stability

ii) $G_{cl} = \frac{2}{s+2+2} = \frac{2}{s+4}$

$\tau = 1/4$, $e_{ss} = 1 - 1/2 = 1/2$

same pole at -4

	i) PI	ii) P	Comparison
speed of response	$\tau = 1/4$	$\tau = 1/4$	Same
accuracy	$e_{ss} = 0$	$e_{ss} = 1/2$	PI better
relative stability	pole at -4	pole at -4	same (both good)

8.

$$G_p(s) = \frac{1}{s^3 + 5s^2 + 6s}$$

$$G_{cl} = \frac{G_p K_p}{1 + G_p K} = \frac{K_p}{s^3 + 5s^2 + 6s + K_p}$$

s^3	1	6
s^2	5	K_p
s^1	$\frac{30 - K_p}{5}$	0
s^0	K_p	

$0 < K_p < 30$

for stability

$e_{ss} = ?$

$y_{ss} = G_{cl}(0) = 1$

$e_{ss} = r_{ss} - y_{ss} = 1 - 1 = 0$