1. a) \( \frac{dy}{dt} + 6y(t) = 4x(t) \)

This is an ordinary differential equation with constant coefficients, therefore, it is linear and time-invariant. It contains memory and it is causal.

b) \( \frac{dy}{dt} + 4y(t) = 2x(t) \)

This is an ordinary differential equation. The coefficients of 4t and 2 do not depend on y or x, so the system is linear. However, the coefficient 4t is not constant, so it is time-varying. The system is also causal and has memory.

c) \( y[n] + 2y[n - 1] = x[n + 1] \)

This is a difference equation with constant coefficients; therefore, it is linear and time-invariant. It is noncausal since the output depends on future values of x. Specifically, let \( x[n] = u[n] \), then \( y[-1] = 1 \).

d) \( y(t) = \sin(x(t)) \)

check linearity:
\[
\begin{align*}
\frac{dy}{dt} + 6y(t) &= \sin(x(t)) \\
\frac{dy_1}{dt} + 6y_1(t) &= \sin(x_1(t)) \\
\frac{dy_2}{dt} + 6y_2(t) &= \sin(x_2(t))
\end{align*}
\]

Solution to an input of \( a_1x_1(t) + a_2x_2(t) \) is \( \sin(a_1x_1(t) + a_2x_2(t)) \).
This is not equal to \( a_1y_1(t) + a_2y_2(t) \).
As a counter example, consider \( x_1(t) = \pi \) and \( x_2(t) = \pi / 2 \), \( a_1 = a_2 = 1 \)

the system is causal since the output does not depend on future values of time, and it is memoryless
the system is time-invariant

the system is causal since the output does not depend on future values of time, and it is memoryless

the system is time-invariant

e) \( \frac{dy}{dt} + y^2(t) = x(t) \)

The coefficient of \( y \) means that this is nonlinear; however, it does not depend explicitly on \( t \), so it is time-invariant. It is causal and has memory.

f) \( y[n + 1] + 4y[n] = 3x[n + 1] - x[n] \)

Rewrite the equation as \( y[n] + 4y[n - 1] = 3x[n] - x[n - 1] \) by decreasing the index.
This is a difference equation with constant coefficients, so it is linear and time-invariant. The output does not depend on future values of the input, so it is causal. It has memory.
3) \( y(t) = \frac{dx}{dt} + x(t) \)

**Linear?**

\[ x_1(t) \mapsto \frac{dx_1}{dt} + x_1(t) = y_1(t) \]
\[ x_2(t) \mapsto \frac{dx_2}{dt} + x_2(t) = y_2(t) \]

\[ a_1 x_1 + a_2 x_2 \mapsto \frac{d}{dt}(a_1 x_1 + a_2 x_2) = a_1 \frac{dx_1}{dt} + a_2 \frac{dx_2}{dt} + a_1 x_1 + a_2 x_2 \]

\[ = a_1 \frac{dx_1}{dt} + a_1 x_1 + a_2 \left( \frac{dx_2}{dt} + x_2 \right) \]

\[ = a_1 y_1 + a_2 y_2 \Rightarrow \text{linear} \]

**Time Invariant?**

\[ x(t) \mapsto y(t) = \frac{dx(t)}{dt} + x(t) \]
\[ x(t-t_1) \mapsto \frac{dx(t-t_1)}{dt} + x(t-t_1) \]

Let \( T = t - t_1 \) and \( dT = dt \)

\[ \frac{dx(T)}{dT} = \frac{dx(T)}{dt} \frac{dt}{dT} = \frac{dx(T)}{dt} \]

So \( \frac{dx(t-t_1)}{dt} + x(t-t_1) = \frac{dx(T)}{dT} + x(T) \)

Now consider \( y(t-t_1) = y(T) = \frac{dx(T)}{dT} + x(T) \)

Given same answer \( \Rightarrow \) **time-invariant**

by inspection, causal & has memory
h) $y[n] = x[2n]$

has memory since the output relies on values of the input at other than the current index $n$.


linear? Let $y_1[n] = x_1[2n]$ and $y_2[n] = x_2[2n]$. The response to an input of $x[n] = ax_1[n] + bx_2[n]$ is

$$y[n] = ax_1[2n] + bx_2[2n],$$

which is $ay_1[2n] + by_2[2n]$, so this is linear.

time-invariant: Let $y_1[n]$ represent the response to an input of $x[n-N]$, so $y_1[n] = x[2(n-N)]$. This is also equal to $y[n-N]$, so the system is time-invariant.

i) $y[n] = nx[2n]$

This is similar to part h), except for the $n$ coefficient. Similar to above, it is noncausal, has memory and is linear. Check time-invariance:

Let $y_1[n]$ represent the response to an input of $x[n-N]$, so $y_1[n] = nx[2(n-N)]$. This is not equal to $y[n-N] = (n-N)x[2(n-N)]$, so the system is time-varying.

j) $\frac{dy}{dt} + \sin(t)y(t) = 4x(t)$

This is an ordinary differential equation with coefficients $\sin(t)$ and 4. Neither depends on $y$ or $x$, so it is linear. However, the explicit dependence on $t$ means that it is time-varying. It is causal and has memory.

k) $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 4y(t) = \frac{dx}{dt} + 4x(t)$

This is an ordinary differential equation with constant coefficients, so it is linear and time-invariant. It is also causal and has memory.

2. The response to $4u(t)$ is $4(1-e^{-2t})u(t)$. The response to $4u(t-1)$ is $4(1-e^{-2(t-1)})u(t-1)$. So the response to $x(t) = 4u(t) - 4u(t-1)$ is $y(t) = 4(1-e^{-2t})u(t) - 4(1-e^{-2(t-1)})u(t-1)$. 