1. a) $\frac{dy}{dt} + 6y(t) = 4x(t)$

This is an ordinary differential equation with constant coefficients, therefore, it is linear and time-invariant. It contains memory and it is causal.

b) $\frac{dy}{dt} + 4ty(t) = 2x(t)$

This is an ordinary differential equation. The coefficients of 4t and 2 do not depend on y or x, so the system is linear. However, the coefficient 4t is not constant, so it is time-varying. The system is also causal and has memory

c)

$$y[n] + 2y[n-1] = x[n+1]$$

This is a difference equation with constant coefficients; therefore, it is linear and time-invariant. It is noncausal since the output depends on future values of x. Specifically, let x[n] = u[n], then y[-1] = 1.

d) y(t) = sin(x(t))

check linearity:

$y_1(t) = \sin(x_1(t))$

$y_2(t) = \sin(x_2(t))$

Solution to an input of $a_1x_1(t) + a_2x_2(t)$ is $\sin(a_1x_1(t) + a_2x_2(t))$.

This is not equal to $a_1y_1(t) + a_2y_2(t)$.

As a counter example, consider $x_1(t) = \pi$ and $x_2(t) = \pi/2$, $a_1 = a_2 = 1$

the system is causal since the output does not depend on future values of time, and it is memoryless
the system is time-invariant

e) $\frac{dy}{dt} + y^2(t) = x(t)$

The coefficient of y means that this is nonlinear; however, it does not depend explicitly on t, so it is time-invariant. It is causal and has memory.

f) $y[n+1] + 4y[n] = 3x[n+1] - x[n]$

Rewrite the equation as $y[n] + 4y[n-1] = 3x[n] - x[n-1]$ by decreasing the index.
This is a difference equation with constant coefficients, so it is linear and time-invariant. The output does not depend on future values of the input, so it is causal. It has memory.

9) $y(t) = \frac{dx}{dt} + x(t)$

linear?

$$x_1(t) \longmapsto \frac{dx_1}{dt} + x_1(t) = y_1(t)$$

$$x_2(t) \longmapsto \frac{dx_2}{dt} + x_2(t) = y_2(t)$$

$$a_1x_1 + a_2x_2 \longmapsto \frac{d(a_1x_1 + x_2a_2)}{dt} + a_1x_1 + a_2x_2$$

$$= a_1\frac{dx_1}{dt} + a_1x_1 + a_2\left(\frac{dx_2}{dt} + x_2\right)$$

$$= a_1y_1 + a_2y_2 \Longrightarrow \text{linear}$$

time invariant?

$$x(t) \longmapsto y(t) = \frac{dx(t)}{dt} + x(t)$$

$$x(t-t_1) \longmapsto \frac{dx(t-t_1)}{dt} + x(t-t_1)$$

let $\tau = t - t_1$

$d\tau = dt$

$$\frac{dx(\tau)}{dt} = \frac{dx(\tau)}{d\tau}\frac{d\tau}{dt} = \frac{dx(\tau)}{d\tau}$$

so $\frac{dx(t-t_1)}{dt} + x(t-t_1) = \frac{dx(\tau)}{d\tau} + x(\tau)$

now consider $y(t-t_1) = y(\tau) = \frac{dx(\tau)}{d\tau} + x(\tau)$

gives same answer $\Longrightarrow$ time-invariant

by inspection, causal & has memory

h) $y[n] = x[2n]$

has memory since the output relies on values of the input at other the the current index n,

causal? Let x[n] = u[n-2], so x[1] = 0. Then y[1] = x[2] = 1, so not causal.

linear? Let $y_1[n] = x_1[2n]$ and $y_2[n] = x_2[2n]$. The response to an input of $x[n] = ax_1[n]+bx_2[n]$ is

$y[n] = ax_1[2n]+bx_2[2n]$, which is $ay_1[2n]+by_2[2n]$, so this is linear

time-invariant: Let $y_1[n]$ represent the response to an input of x[n-N], so $y_1[n] = x[2(n-N)]$. This is also equal to y[n-N], so the system is time-invariant.

i) $y[n] = nx[2n]$

This is similar to part h), except for the n coefficient. Similar to above, it is noncausal, has memory and is linear. Check time-invariance:

Let $y_1[n]$ represent the response to an input of x[n-N], so $y_1[n] = nx[2(n-N)]$. This is not equal to y[n-N] = (n-N)x[2(n-N)], so the system is time-varying.

j) $\frac{dy}{dt} + \sin(t)y(t) = 4x(t)$

This is an ordinary differential equation with coefficients sin(t) and 4. Neither depends on y or x, so it is linear. However, the explicit dependence on t means that it is time-varying. It is causal and has memory.

k) $\frac{d^2y}{dt^2} + 10\frac{dy}{dt} + 4y(t) = \frac{dx}{dt} + 4x(t)$

This is an ordinary differntial equation with constant coefficients, so it is linear and time-invariant. It is also causal and has memory.

2. The response to 4u(t) is $4(1-e^{-2t})u(t)$. The response to 4u(t-1) is $4(1-e^{-2(t-1)})u(t-1)$. So the response to x(t) = 4u(t) - 4u(t-1) is $y(t) = 4(1-e^{-2t})u(t) - 4(1-e^{-2(t-1)})u(t-1)$.