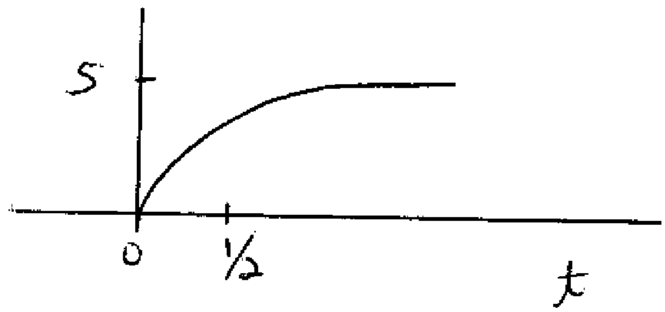
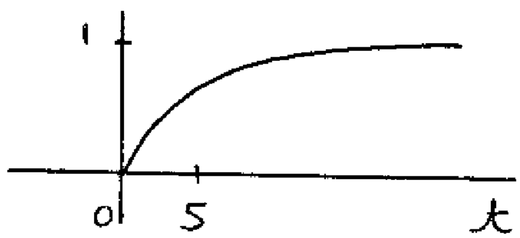
1. a) $H(s) = \dfrac{10}{s+2}$

$y_{ss} = \lim_{s \to 0} H(s) = 5$, $\tau = 1/2$ sec

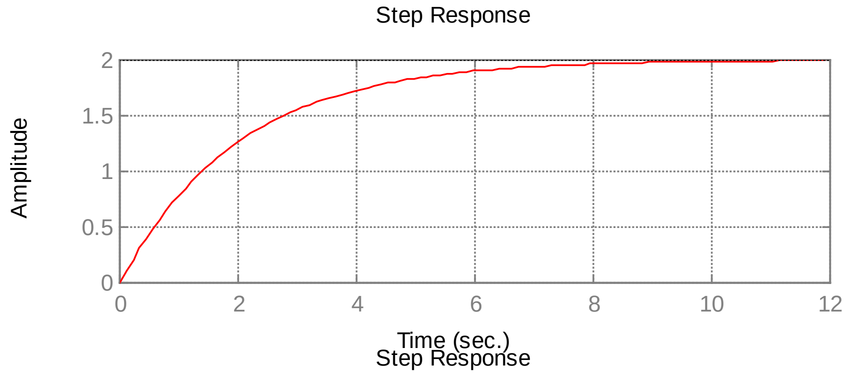


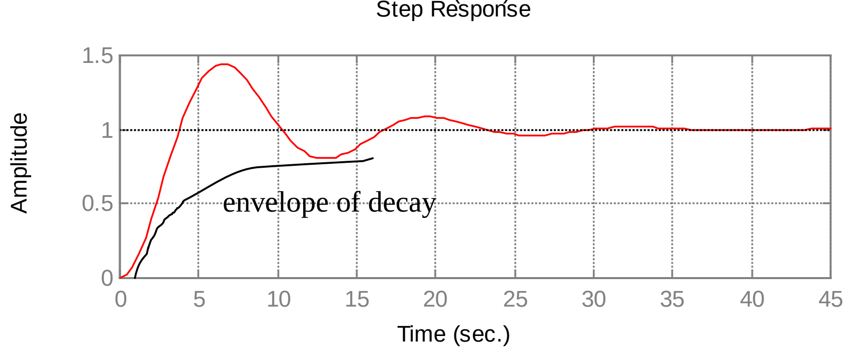
b) $H(s) = \dfrac{0.2}{s + 0.2}$

$y_{ss} = 1$, $\tau = 1/0.2 = 5$ sec



2.





First order system (top plot) has general form $H(s) = \frac{k}{s+a}$. The time-constant is the time that the response is equal to $2(1-e^{-1}) = 63\%$ of $2 = 1.26$, so $\tau \approx 2$ sec. $a=1/\tau = 0.5$. The steady-state value (due to a unit step input) is $H(0) = k/a = k/0.5$. From the plot $2 = k/0.5$ so $k = 1$.

Final answer: $H(s) = \frac{1}{s+0.5}$

Second order system (bottom plot) has general form $H(s) = \frac{k}{s^2 + 2\zeta\omega_n s + \omega_n{}^2}$ or

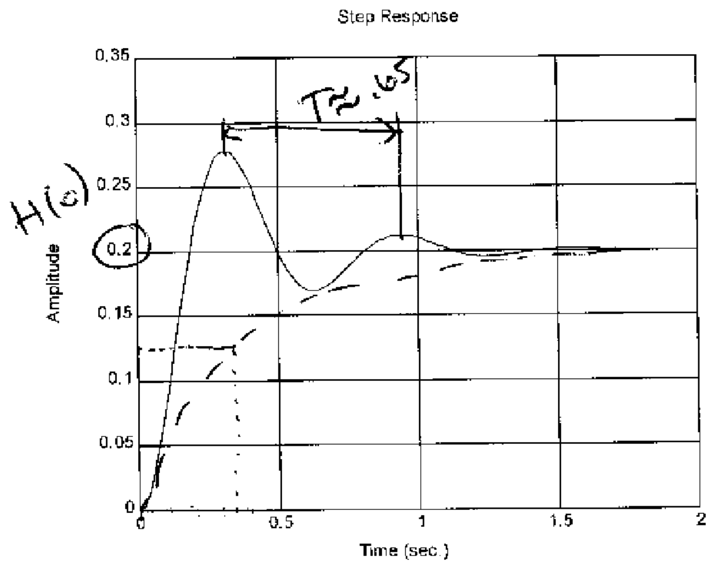
$H(s) = \frac{k}{(s+\zeta\omega_n)^2 + \omega_d{}^2}$ where the real part of the pole is at $-\zeta\omega_n$ is the real part of the pole (it governs the envelope of decay) and ω_d is the imaginary part of the pole (it governs the frequency of the oscillations, $\omega_d = 2\pi/T$).

From the plot, $T \approx 12$ sec, so $\omega_d = 2\pi/13$. The time constant of the envelope of decay is about $\tau \approx 7$ sec, so $\zeta\omega_n=1/7$. k is found from the steady-state value $1 = H(0) = \frac{k}{(\zeta\omega_n)^2 + \omega_d{}^2}$. Solving fo k yields $k \approx 0.254$.

Final answer: $H(s) = \frac{0.254}{s^2 + 0.286s + 0.254}$

c)

$$H(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$



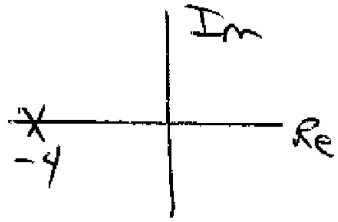
$$\omega_d = \frac{2\pi}{T} = 9.7$$

$$\tau \approx .35 \text{ sec} \Rightarrow \frac{1}{\zeta\omega_n} = \tau \Rightarrow \zeta\omega_n = 2.9$$

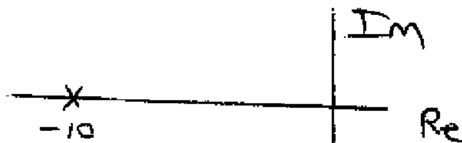
$$H(s) = \frac{K}{(s+2.9)^2 + 9.7^2}, \quad H(0) = 0.2 \Rightarrow K = 20.5$$

$H(s) = \dfrac{20.5}{(s+2.9)^2 + 9.7^2}$ from plot. Actual is $H(s) = \dfrac{22}{(s+3)^2 + 10^2}$

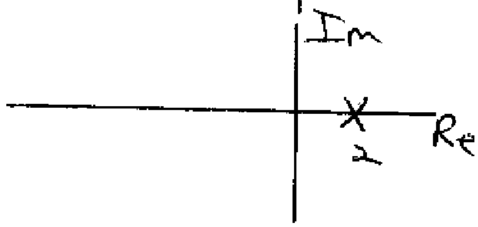
3 a) $H(s) = \dfrac{1}{s+4}$



b) $H(s) = \dfrac{1}{s+10}$



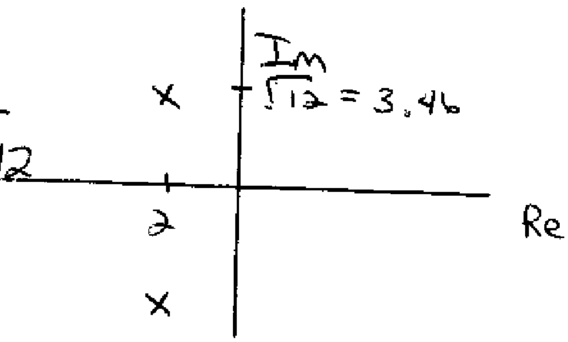
c) $H(s) = \dfrac{1}{s-2}$



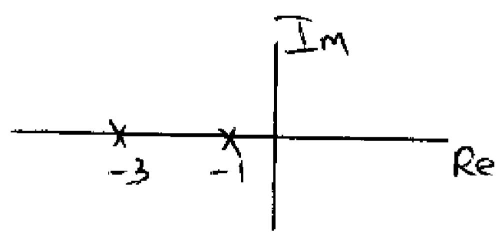
d) $H(s) = \dfrac{1}{s^2+4s+16} = \dfrac{1}{(s+2)^2+12}$

$\omega_n^2 = 16 \Rightarrow \omega_n = 4$

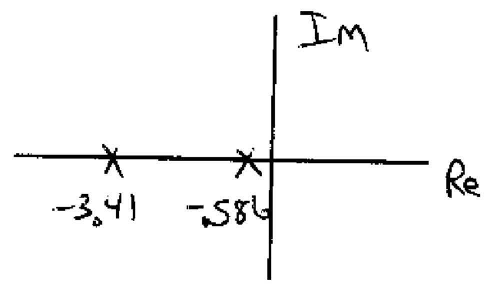
$2\zeta\omega_n = 4 \Rightarrow \zeta = 0.5$



e) $H(s) = \dfrac{1}{s^2+4s+3} = \dfrac{1}{(s+1)(s+3)}$



f) $H(s) = \dfrac{1}{s^2+4s+2} = \dfrac{1}{(s+3.41)(s+0.586)}$

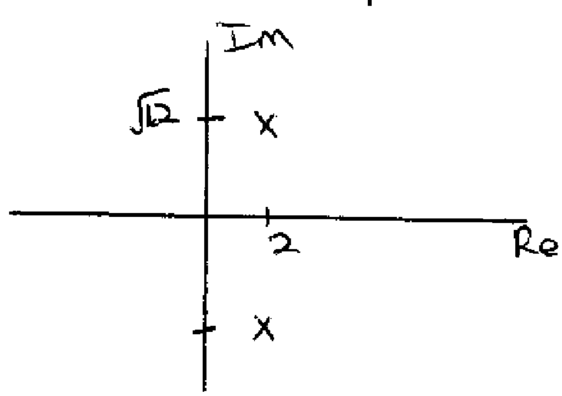


g) $H(s) = \dfrac{1}{s^2-4s+16}$, $\omega_n = 4$

$2\zeta\omega_n = -4$

so $\zeta = -0.5$

$= \dfrac{1}{(s-2)^2+12}$



4\. a)

$$y(t) = k_1 + k_2 e^{-4t}, \quad t \geq 0$$

(note, k_1 is found from steady-state $= H(0) = \frac{1}{4}$)

b) $y(t) = k_1 + k_2 e^{-10t}, \quad t \geq 0$

c) $y(t) = k_1 + k_2 e^{2t}, \quad t \geq 0$

d) $y(t) = k_1 + k_2 e^{-2t} \cos(\sqrt{12}\, t + \theta), \quad t \geq 0$

e) $y(t) = k_1 + k_2 e^{-t} - k_3 e^{-3t}, \quad t \geq 0$

f) $y(t) = k_1 + k_2 e^{-3.41t} + k_3 e^{-0.586t}, \quad t \geq 0$

g) $y(t) = k_1 + k_2 e^{2t} \cos(\sqrt{12}\, t + \theta), \quad t \geq 0$

in all cases, $k_1 = H(0)$

5.

$$A\cos(\omega t+\theta)u(t) \longmapsto y_t(t) + y_{ss}(t)$$

$$A\,|H(\omega)|\cos(\omega t+\theta+\angle H(\omega))$$

let $x(t) = 2\cos(4t-20^\circ)\,u(t)$

a) $H(s) = \frac{1}{s+4}$ so $H(\omega) = \frac{1}{j\omega+4}$

$$H(4) = \frac{1}{4j+4} = 0.176\,e^{-45^\circ j}$$

$$y_{ss}(t) = 0.352\cos(4t-20^\circ-45^\circ)$$
$$= 0.352\cos(4t-65^\circ)$$

) $H(s) = \frac{1}{s^2+4s+16}$, $H(\omega) = \frac{1}{(16-\omega^2)+4j\omega}$

$$H(4) = \frac{1}{16j} = 0.0625\,e^{-j90^\circ}$$

$$y_{ss}(t) = 0.125\cos(4t-110^\circ)$$

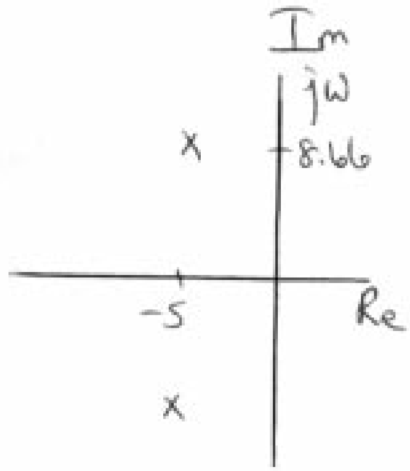
f) $H(s) = \frac{1}{s^2+4s+2}$, $H(\omega) = \frac{1}{(2-\omega^2)+4j\omega}$

$$H(4) = \frac{1}{-14+16j} = 0.047\,e^{-j131^\circ}$$

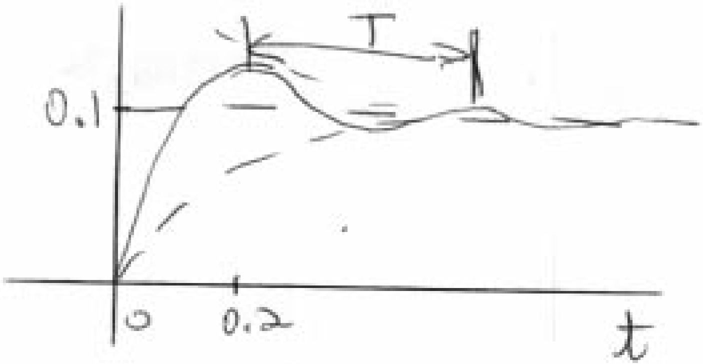
$$y_{ss}(t) = 0.094\cos(4t-151^\circ)$$

a) $W_n = 10,\quad 2\zeta\omega_n = 10 \longrightarrow \zeta = 0.5$

$$P_1, P_2 = -5 \pm 8.66j$$



b) $\tau = \frac{1}{\zeta\omega_n} = \frac{1}{5},\quad T = \frac{2\pi}{\omega_d} = 0.726$



$y_{ss} = H(0) = 0.1$

c) $$H(10) = \frac{10}{(j10)^2 + 10j\omega + 100} = \frac{10}{100j} = 0.1e^{-j\pi/2}$$

$$y_{ss}(t) = 0.1\cos(10t - \pi/2)$$