DTFT and DFT:

1. Compute the DTFT of the following signals and sketch $X(\omega)$.
a) $x[n] = [1/4\ 1/4\ 1/4\ 1/4]$
b) $x[n] = [1\ \text{-}2\ 1]$

c) $x[n] = 2\left(\dfrac{3}{4}\right)^n u[n]$

2. Compute by hand the DFT of the signals given in Problem 1 a) and b) and compare your answers to those found in Problem 1.

3. Use MATLAB to compute the DFT of the signals in Problem 2. In order to increase your accuracy, pad the signals with zeros (for example. if you want to plot an additional 10 points, then add 10 zeros). For the signal in part c), compute the DFT for three cases: truncating the signal at $N = 5$, $N = 10$ and at $N = 20$. In each case, compare your answers to those found in Problem 1 by plotting the magnitude of both versus frequency.

5. Find the DTFT and the DFT of $x[n] = [1\ \text{-}1]$. Sketch $X(\Omega)$ for $\text{-}\pi < \Omega < \pi$.

6. The DTFT for an 6 point signal, $x[n]$, is shown below. Indicate on the plot the values of the DFT.
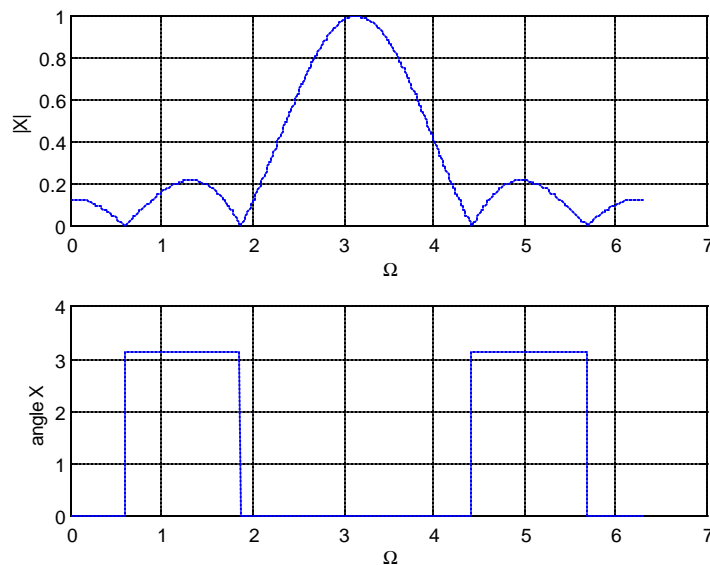
7. Use the FFT to approximate the Fourier Transform of $x(t) = 4e^{-t}u(t)$. Consider the following cases:

i) Sampling period $T = 1$, $N = 10$.
ii) Sampling period $T = 1$, $N = 20$.
iii) Sampling period $T = 0.5$, $N = 20$.
iv) Sampling period $T = 0.1$, $N = 100$.

Use MATLAB to compute and plot the approximations of $|X(\omega)|$ for each of these cases (use the symbol 'o' to denote the approximation) along with the actual $|X(\omega)|$ for comparison.

8. Consider the continuous-time signal $x(t) = 2e^{-2t}u(t)$. You wish to use the FFT to approximate $X(\omega)$.

a) Determine the frequency ω_B such that $|X(\omega)| < 0.02|X(0)|$. The frequency content of $x(t)$ is negligible above this value.

b) Determine an appropriate minimum value for the sampling period T from the information determined in part a).

c) Using the value of T determined in part b), determine the number of points N of $x(t)$ to be sampled so that the resolution of the approximation from the FFT is $\Gamma = 0.2$ rad/sec.

d) Sketch $|X(\omega)|$ and your approximation of it obtained using the FFT. (You do not need to actually compute the approximation, just estimate how it might look.)

e) Use MATLAB to compute and plot the approximation along with the actual plot of $|X(\omega)|$.

9. Compute the DTFT and the DFT of the following signal.

$$x[n] = [1\ 0\ 1]$$

Sketch the DTFT $|X(\Omega)|$ and mark where on the plot the DFT terms are.

10. Suppose that you wish to approximate the Fourier Transform of a continuous-time signal,

$$x(t) = e^{-2t}\cos(10t)u(t)$$

Plot $|X(\omega)|$. If $T = 0.1$ sec, pick N to get a resolution on the frequency scale of $\Gamma \leq 0.1$ rad/sec.