Final Project

Due 3/11/99
Form 2 person groups, and turn in one report from each group. Your report should be presented neatly and be well-organized. Include an introduction that gives a short description of the problem, a description of your design methods (including all derivations), all plots and schematics (adequately labeled and described).

A processing plant has two tanks that are connected as shown in the figure. The controls engineer would like to design a control system to regulate the level of Tank 2, \( h_2(t) \), by modifying the input flow \( x(t) \) into Tank 1. Note that there is a flow out of the tanks. The engineer installs photodiodes in Tank 2 to detect the fluid level of the tank. Hence, let \( y(t) = h_2(t) \) be the variable available for feedback.

In this system model, there is a constant input flow \( x_o \) into Tank 1 that balances the constant flow \( x_o \) out of Tank 2. Without any additional input from the feedback controller (i.e., \( x(t) = 0 \)), the levels in the tank reach steady-state values of \( H_1 \) and \( H_2 \). The equations given below are written to only reflect the variation from these constant values. That is, if the level of Tank 1 is higher than \( H_1 \), then \( h_1(t) \) is positive. The equations for the system are given by:

\[
\begin{align*}
  x_1(t) &= \frac{h_1(t) - h_2(t)}{R_1} \\
  x(t) - x_1(t) &= C_1 \frac{dh_1}{dt} \\
  x_2(t) &= \frac{h_2(t)}{R_2} \\
  x_1(t) - x_2(t) &= C_2 \frac{dh_2}{dt}
\end{align*}
\]

The parameters \( C \) and \( R \) are called capacitance and resistance of the tanks. Let \( C_1 = 2 \text{ m}^2 \), \( C_2 = 4 \text{ m}^2 \), \( R_1 = 0.2 \text{ sec/m}^2 \), \( R_2 = 0.4 \text{ sec/m}^2 \).

The goal is to design a controller, \( G_c(s) \), that measures the level of Tank 2, compares it to a reference \( r(t) \), and inputs the error into a compensating circuit. The compensating circuit controls a valve that can increase or decrease \( x(t) \). Accordingly, let \( E(s) = R(s) - Y(s) \) be the input to the compensator and let \( X(s) \)
be the output. The design specifications for the closed loop system in terms of step response \( r(t) = u(t) \) are: time constant no more than 0.5 seconds, steady-state error no more than 7% (i.e., 0.07 for a unit step input), and percent overshoot of no more than 20%. (This is calculated by \( PO = 100(y_{\text{max}} - y_{ss})/y_{ss}. \) MATLAB has a command \text{max} that will calculate the maximum value of a vector.)

1. Build a block diagram of the system. First build components to fit each equation, then put the components together so that \( Y(s) \) is the output and \( X(s) \) is the input. Reduce the block diagram to get the input/output transfer function, \( G_p(s) \).

2. Design three controllers to satisfy (as best possible) the criteria: a proportional controller, a PI controller and a PID controller. For each case, determine the closed loop pole positions and the resulting values for \( \tau, PO, \) and \( e_{ss} \). You can play around with the zero locations for the PI and the PID controllers. Plot the closed loop responses to \( r(t)=u(t) \) for each controller.

3. Next, you need to design compensating circuits that implement the controllers. A proportional controller is implemented with an amplifier, but the PI and PID need compensating circuits. The circuit diagrams for the PI and the PID are given below. Use circuit equations to obtain a transfer function for each circuit. Note that an ideal op amp has the characteristics that the voltage drop across the input terminals is zero and the current into the op amp is also zero. Select resistance and capacitance values (not to be confused with the resistances and capacitances of the fluid system) so that the circuit transfer functions are equal to \( G_c(s) \) for PI and for the PID compensators designed in Part 2. For practical reasons, limit your values to \( 1000\Omega < R < 1M\Omega \) and \( 0.1\mu f < C < 500\mu f \).

![PI Circuit](image)

![PID Circuit](image)