1. a) \[ Y(s) = C_p(s) C_c(s) \frac{C_d(s) 10}{S + 1} \]

need pole at -2 to have \( T = 0.5 \)

let \( C_c(s) = \frac{(s+1) K}{s+2} \)

\[ Y(s) = \frac{1}{s+2} \frac{K10}{s} \]

\[ y_{ss} = \lim_{s \to 0} sY(s) = \frac{K10}{2} \]

\[ C_c(s) = \frac{(s+1)2}{s+2} \]

b) \[ \frac{Y(s)}{R(s)} = \frac{1 - (s+1)2}{s+5} \frac{s+2}{s+2} \]

\( T \) is governed by the slowest pole
so \( T = \frac{1}{\beta_S} = 2 \) sec

\[ Y(s) = \frac{(s+1)2}{(s+5)(s+2)} \frac{10}{s} \]

so \( y_{ss} = 20 \)

\[ \Rightarrow e_{ss} = r_{ss} - y_{ss} = -10 \]
2. a) \[ \frac{Y(s)}{R(s)} = \frac{G_c G_m}{1 + G_c G_m} \]
\[ = \frac{K_p}{s + 1} = \frac{K_p}{s + K_p} \]

pick \( K_p = 1 \)

b) \[ y_{ss} = \lim_{s \to 0} sY(s) = \frac{10}{2} \] so \( e_{ss} = r_{ss} - y_{ss} = 5 \)

c) 
\[ y(t) \]
\[
\begin{array}{c}
1 \\
\alpha \\
5 \\
0 \\
t
\end{array}
\]

d) \( K_p = 4 \) \[ \Rightarrow \frac{Y(s)}{R(s)} = \frac{4}{s + 5} \]
\[ \tau = \frac{1}{5} \text{ sec} \]
\[ y_{ss} = \lim_{s \to 0} sY(s) = \frac{4}{5 + 5} \bigg|_{s=0} = 8 \]
\[ e_{ss} = 2 \]

\[ y(t) \]
\[
\begin{array}{c}
1 \\
1 \\
8 \\
0 \\
\frac{1}{5} \\
t
\end{array}
\]
e) The plots below are the closed loop step responses for the controllers defined in a) and d) (with the nominal plant). Top plot is for $K_P = 1$ and the bottom plot is for $K_P = 4$. 

![Step Response Graphs]
f) The actual closed loop system is represented by

\[ G_{cl}'(s) = \frac{G_c G_p'}{1 + G_c G_p'} = \frac{K_p}{1 + K_p + 0.5} \]

where \( G_p' \) represents the actual plot. The closed loop step responses are shown for \( K_p = 1 \) (top plot) and \( K_p = 4 \) (bottom plot).
3. \( G_p = \frac{1}{s + 0.5} \), let \( R(s) = \frac{R_o}{s} \) \((r(t) = R_o u(t))\)

\[ Y(s) = \frac{4}{s + 5} \frac{R_o}{s} \]

\[ y_{ss} = \frac{4R_o}{5} = 10 \Rightarrow R_o = \frac{50}{4} = 25 \]

Apply \( r(t) = \frac{25}{2} u(t) \) to actual plant

\[ Y(s) = \frac{1}{s + 4.5} \frac{25}{2s} \]

\[ y_{ss} = \frac{50}{4.5} = 11.11 \]

\[ e_{ss} = r_{ss} - y_{ss} = -1.11 \]
4. \[ G_c = \frac{K_p (s+1)}{s} \]

\[ G_{cd} = \frac{G_c G_p}{1+G_c G_p} = \frac{K_p s}{1+K_p s} = \frac{K_p}{s+K_p} \]

a). \[ K_p = 2 \]

b). \[ y(s) = \frac{K_p}{s+K_p} \cdot \frac{10}{s}, \text{ if } K_p = 2 \quad y_{ss} = 10 \]

so \[ e_{ss} = r_{ss} - y_{ss} = 0 \]

d) \[ \text{num} = 2; \]
\[ \text{den} = [1, 2]; \]
\[ \text{step (num * 10, den)} \]
\[ \text{title ('PI for nominal plant')} \] (see plot next page)

e) \[ G_{cd} = \frac{G_c G_p}{1+G_c G_p} = \frac{2(s+1)}{s} \cdot \frac{1}{1+\frac{2(s+1)}{s(s+5)}} = \frac{2(s+1)}{s^2 + 2.5s + 2} \]

\[ \text{num} = [2, 2]; \quad \text{den} = [1, 2.5, 2]; \]
\[ \text{step (10 * num, den)} \]
f) The following plots show the closed loop step response for the nominal plant with the PI compensator (top plot) and the actual plant with the PI compensator (bottom plot).
5. Comparison of closed loop results:

<table>
<thead>
<tr>
<th></th>
<th>$e_{ss}$ (nominal plant)</th>
<th>$\tau$ (nominal plant)</th>
<th>$e_{ss}$ for actual plant</th>
<th>$\tau$ (nominal plant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open Loop</td>
<td>0</td>
<td>0.5</td>
<td>-10</td>
<td>2</td>
</tr>
<tr>
<td>$P$ with $K_P = 1$</td>
<td>5</td>
<td>0.5</td>
<td>3.33</td>
<td>0.667</td>
</tr>
<tr>
<td>$P$ with $K_P = 4$</td>
<td>2</td>
<td>0.2</td>
<td>1.11</td>
<td>0.22</td>
</tr>
<tr>
<td>$P$ with open loop adj.</td>
<td>0</td>
<td>0.2</td>
<td>-1.11</td>
<td>0.22</td>
</tr>
<tr>
<td>PI</td>
<td>0</td>
<td>0.5</td>
<td>0</td>
<td>0.8</td>
</tr>
</tbody>
</table>

For nominal behavior, the open loop, the $P$ with larger value of $K_P$ (especially with the open loop adjustment in $r(t)$) and the PI all work well. All can be made faster by proper choice of the controller parameters.

You can get a measure of robustness by noting how sensitive the controller is to the change in the plant. For example, how close is the behavior of the closed loop system with the actual plant to that of the closed loop system with the nominal plant. The open loop controller is very sensitive with very large changes in $\tau$ and $e_{ss}$. The $P$ controller with larger gain is less sensitive than the $P$ with smaller gain (the percent change in the $\tau$ and $e_{ss}$ are lower). The PI controller is not sensitive at all with respect to steady-state error, but is somewhat sensitive to the time constant.