Problem 1 (20 Points):

a) Give the general form of the expression for \( y(t) \).

\[
Y(s) = \frac{20(s + 1)}{(s^2 + 16)((s + 4)^2 + 25)(s+1)}
\]

\( y(t) = k_1 \cos(4t + \theta_1) + k_2 e^{-\frac{t}{4}} \cos((s-t)+\theta_2) + k_3 e^{-t} \quad t \geq 0 
\)

b) Find the range of \( K \) for the system to be stable:

\[
H(s) = \frac{10 + 2}{s^3 + 3s^2 + 4s + K}
\]

\[
\begin{array}{c|ccc}
  s^3 & 1 & 4 \\
  s^2 & 3 & K \\
  s & \frac{12-K}{3} & 0 \\
  s^0 & K \\
\end{array}
\]

\( 0 < K < 12 \)
Problem 2 (20 Points):

Find the transfer function of the following circuit when $R_1 = R_2 = 1000\Omega$ and $C = 100\mu F$. Is the system stable?

\[
X(s) - \frac{1}{R + CS} = \frac{R}{1 + RCS}
\]

\[
Y(s) = X(s) \frac{R}{1 + RCS} = \frac{R}{R + \frac{B}{1 + RCS}}
\]

\[
H(s) = \frac{R}{R + RCS + R}
\]

\[
= \frac{1000}{2000 + 100s} = \frac{10}{s + 20}
\]

This is stable since pole is at $-20$. 

By voltage divider law
Problem 3 (35 Points):

Solve the differential equation using the Laplace Transform method.

\[ y'' + 7y' + 12y = 6x, \quad x(t) = u(t), \quad y(0) = 2; \quad y'(0) = 0 \]

\[ s^2 Y(s) - sy(0) - y'(0) + 7sY(s) - 7y(0) + 12Y(s) = \frac{6}{s} \]

\[ (s^2 + 7s + 12) Y(s) = \frac{6}{s} + 2 \]

\[ Y(s) = \frac{6}{s(s+4)(s+3)} + \frac{2}{(s+4)(s+3)} \]

\[ Y(s) = \frac{1}{s} + \frac{3/2}{s+4} + \frac{-2}{s+3} \]

\[ y(t) = \frac{1}{2} + \frac{3}{2} e^{-4t} - 2 e^{-3t} - 2e^{-4t} + 2e^{-3t}, \quad t \geq 0 \]

\[ = \frac{1}{2} - \frac{1}{2} e^{-4t}, \quad t \geq 0 \]
Problem 4 (25 Points):

a) Reduce the block diagram to one block

\[ X(s) \rightarrow H_1(s) \rightarrow H_3(s) \rightarrow Y(s) \]
\[ \downarrow \quad \downarrow \quad \downarrow \]
\[ + \quad + \quad + \]
\[ H_2(s) \quad H_4(s) \quad \]

b) Explain why block diagrams are important. It is easier to model smaller subsystems and use block diagram reduction than to try to work with all the equations for the entire system all at once.

\[ X(s) \rightarrow \frac{H_1}{1+H_1H_2} \rightarrow H_3 + H_4 \rightarrow Y(s) \]
\[ \rightarrow \]

\[ X(s) \rightarrow \frac{H_1(H_3 + H_4)}{1+H_1H_2} \rightarrow Y(s) \]