1. A two tank reactor system is shown in the block diagram. It is required to maintain the concentration of liquid in the second tank at a desired level, in spite of variation of inlet concentration to the first tank, by the addition of reactant through a control valve. A linearized model is obtained where $r=0$ = deviation in desired concentration; $x_1$ = deviation in concentration of second tank; $x_2$ = deviation in concentration of first tank; $u$ = actuator control value signal. The actuator block has fast dynamics compared to the rest of the system and can be represented through scaling the complex frequency $s$ by $\varepsilon = 1/4$ as

$$\frac{1}{(\varepsilon s)^2 + 2\varepsilon s + 1}$$

a) Write the system in state space form so that singular perturbation can be applied. Let $x = [x_1 \ x_2]^T$ and $z = [z_1 \ z_2]^T$ (where $z_2 = \varepsilon dz_1/dt$).

b) Design a feedback for the slow subsystem to place the poles at $-0.707 \pm 0.707j$. Plot the free response to initial conditions $x(0) = [2.5 \ 2]^T$ and $z(0) = [0 \ 0]^T$ for the controlled slow model and for the controlled full order system.
2. The control of flexible structures has received a lot of attention in the past ten years. One application is the control of antenna for accurate pointing. These structures are typically comprised of numerous flexible truss elements interconnected by joints. One of the control strategies investigated is active joints which can damp out flexible modes. As an example of this, look at the two beam system shown in the figure which has a motor at the joint which is used for control. The figure also shows a disturbance term f(t). The model is of the form:

\[
\begin{align*}
\dot{x} &= \begin{bmatrix}
0 & \mathbf{I}_6 \\
-\Omega^2 & -2\zeta\Omega
\end{bmatrix} x + \begin{bmatrix}
0 \\
0
\end{bmatrix} f + \begin{bmatrix}
0 \\
0
\end{bmatrix} u \\
y &= \begin{bmatrix}
1.58 & 2.2 & 1.58 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} x \\
&\quad \begin{bmatrix}
0 & 0 & 0 & 1.2 & -1.76 & 1.2 & 0 & 0 & 0 & 0
\end{bmatrix} x
\end{align*}
\]

where \( x \in \mathbb{R}^{12}, u \in \mathbb{R}, y \in \mathbb{R}^2 \)

\[
\Phi = [-7 \ 14 \ -21 \ 2.9 \ 5.8 \ 8.7]^T
\]

\[
\Omega = \text{diag}\{69.8, \ 279, \ 628, \ 21.5, \ 86.1, \ 64.6\}
\]

\[
\Gamma = [1.6 \ 2.2 \ 1.6 \ 0 \ 0 \ 0]^T
\]

Let \( \zeta = 0.1 \).

a) Perform balanced model reduction on the system to obtain models that are 4th, 6th order, and 10th order. Note, to recover the behavior due to the disturbance, treat the disturbance as an additional input for the sake of the model reduction. Compare these reduced order models via the response to a simultaneous impulse in \( f(t) \) and \( u(t) \). (You can use superposition to get this result.) In addition, compute a 4th order model obtained without considering the disturbance.

b) Design two output feedback controllers based on the two 4th order models obtained in part a). You want the system to behave as if it had damping of 0.4. Simulate the impulse response due to \( f(t) = \delta(t) \). Simulate the full order model with these controls and compare the results to the controlled reduced order models.