Quiz Solution

Given the 8-ary QAM constellation

(a) Give the total energy by summing the energy of all the signals $s_m(t)$ for $m=1,...,8$. Assume $E_g=0.01$ and assume the scalar factor in the amplitudes is $d = 0.5$.

(b) Move four signal vectors and redraw the constellation for the 8-ary QAM that would lessen the total energy and maintain at least the minimum distance between points as shown in the constellation shown above.

(c) Does the constellation shown above represent a simplex set?

Solution:

(a) $E_m = \|s_m\|^2 = \frac{E_g}{2} \left( A_{m_1}^2 + A_{m_2}^2 \right) = \frac{E_g}{2} d^2 (a_{ms}^2 + a_{mc}^2)$

where $a_{ms} \in \{\pm 1\}$, $a_{mc} \in \{\pm 1, \pm 3\}$

$E_T = \sum_{m=1}^{8} E_m = \frac{E_g}{2} d^2 (2 + 2 + 2 + 2 + 10 + 10 + 10 + 10)$

$= 0.005 \times 0.25 \times 48 = 0.06$

(b) Move the 4 most distant points to the axes as shown below. You can maintain the same minimum distance between points, but the distance of the further points to the origin is now less than it was, so the overall energy has been reduced.

(c) No for several reasons. Some of the properties of a simplex set are that the vectors have zero mean (they sum to zero, which is true in this case), have equal energy (not true in this case), and have equal spacing (not true in this case since each signal vector is farther from some than others). In addition, there was mention in the book that the dimension of the space is $M-1$ (not true in this case since this is a 2 dimensional space while $M=8$). This property stems from the fact that the simplex sets starts with $M$ orthogonal vectors and translates them. It can be shown that this particular translation removes one dimension. You could further rotate the vectors to plot them in a smaller dimension. Thus, to have a simplex set in 2 dimensions, you would have 3 signal vectors that are equally spaced and are centered around the origin.